

การเขียนหนังสือภาษาไทยด้วย L^AT_EX

ทวีศักดิ์ สمانชื่น

PUBLISHER

Copyright © 2020 ทวีศักดิ์ สมานชื่น

<cfŠTMr05IDA

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying and recording, or by any information storage or retrieval system, without the prior written permission of the publisher.

Art. No xxxxx

ISBN xxx-xx-xxxx-xx-x

Edition 0.0

Cover design by Cover Designer

Published by Publisher

Printed in City

Ö-ö ð ð =Ztt Yzç ÷ ù ð ñ ö {tD{5ccX ùö
ù ö ùö ð ð ð ð ù ñ ñö
Ic_{ ð ð ö ð ð Z{TM =cAD ö ÷ ö
ð ù ùð ùöð ðññ ó ÷ñö-Ztt ó ó
ð ð ð ù Yzç ó ù ð ð
ùö ð ó ñ ó ð
ð ù

Propositional Logic

\neg	$\neg P$	5
\wedge	$P \wedge Q$	6
\vee	$P \vee Q$	7

Let P and Q be logical propositions (true or false). Then the following propositions are true:

1.1 First section

Let's start out with the following theorem.

Theorem 1.1 Let P , Q and R be logical propositions (true or false). Then the following propositions are true:

$P \wedge Q, Q \wedge P$	$P \vee Q, Q \vee P$	(commutative laws)
$(P \wedge Q) \wedge R, P \wedge (Q \wedge R)$	$(P \vee Q) \vee R, P \vee (Q \vee R)$	(associative laws)
$P \wedge (Q \vee R), (P \wedge Q) \vee (P \wedge R)$	$P \vee (Q \wedge R), (P \vee Q) \wedge (P \vee R)$	(distributive laws)
$\neg(P \wedge Q), \neg P \vee \neg Q$	$\neg(P \vee Q), \neg P \wedge \neg Q$	(De Morgan's laws)

We prove the first of De Morgan's laws and leave the proofs of the remaining propositions as exercises. To prove the statement, we create a truth table and fill in all possible values (true or false) for the propositions P and Q . Each of these propositions can be either true or false and we thus obtain the following truth table with four cases:

$(P \wedge Q)$	$P \rightarrow Q$
T	T
T	F
F	T
F	F

By definition of the logical operators, we complete the table to obtain

$(P \wedge Q)$	$P \rightarrow Q$
F	T
T	F
T	T
F	F

It follows that the statement we want to prove (the equivalence,) is always true (a $\{ \in \mathbb{Z} \}$), which proves the statement. □

1.2 Second section

We begin our next section with the following central definition.

Definition 1.1 A rational Cauchy sequence is a rational sequence $(x_n)_{n=0}^{\infty}$ such that

$$\forall \epsilon > 0 \exists N \in \mathbb{N} : \forall m, n \geq N, |x_m - x_n| < \epsilon \tag{1.1}$$

In other words, for each (small) rational number $\epsilon > 0$ there is a (big) number N such that the distance $|x_m - x_n|$ between x_m and x_n is less than ϵ if both m and n are larger than or equal to N .

Definition 1.2 A real number is a Cauchy sequence of rational numbers that converges to a limit in \mathbb{R} .

Example 1.1 Consider the equation $x^2 = 2$. It is easy to prove that this equation does not have any rational solutions. However, consider the following iteration formula:

$$x_{n+1} = \frac{x_n + \frac{2}{x_n}}{2}; \tag{1.2}$$

where $n = 1; 2; 3; \dots$ and $x_0 = 1$. The resulting sequence of rational numbers quickly

approaches a number in the vicinity of $x = 1.4142135623731$:

$$\begin{aligned}
 x_0 &= 1 \\
 x_1 &= (x_0 + 2/x_0)/2 = 1.5 \\
 x_2 &= (x_1 + 2/x_1)/2 = 1.4166666666667 \\
 x_3 &= (x_2 + 2/x_2)/2 = 1.4142156862745 \\
 x_4 &= (x_3 + 2/x_3)/2 = 1.4142135623747 \\
 x_5 &= (x_4 + 2/x_4)/2 = 1.4142135623731 \\
 x_6 &= (x_5 + 2/x_5)/2 = 1.4142135623731 \\
 x_7 &= (x_6 + 2/x_6)/2 = 1.4142135623731 \\
 x_8 &= (x_7 + 2/x_7)/2 = 1.4142135623731 \\
 x_9 &= (x_8 + 2/x_8)/2 = 1.4142135623731 \\
 x_{10} &= (x_9 + 2/x_9)/2 = 1.4142135623731
 \end{aligned}$$

We will later see that this iteration, or any other equivalent iteration, defines the real number $\sqrt{2}$.

1.3 Third section

Now let's move on to the definition of the real number system. This may be defined in a multitude of ways, one of which is to think about a real number as a rational Cauchy sequence, or rather the equivalence class of Cauchy sequences "converging to" that number.

The real numbers \mathbb{R} is the set of all equivalence classes of rational Cauchy sequences.

Now that this is settled, let's prove the completeness of the real number system.

Let $(x_n)_{n=0}^{\infty}$ be a sequence of real numbers. Then $(x_n)_{n=0}^{\infty}$ is convergent if and only if it is also a real Cauchy sequence.

Write $x_m = [(x_{mn})_{n=0}^{\infty}]$ where x_{mn} is the n th number in a rational Cauchy sequence representing the real number x_m . And so on.... \square

For further reading, there are several excellent works that one could cite, such as [1, 2].

Exercises

Exercise 1.3.1 Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Determine the following sets.

- (a) $A \cup B$ (b) $A \setminus B$ (c) $A \cap B$ (d) $A \setminus A$

Exercise 1.3.2 Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$. Determine the following sets.

- (a) $A \cup B$ (b) $A \setminus B$ (c) $A \cap B$ (d) $A \setminus A$

Exercise 1.3.3 Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$. Determine the following sets.

- (a) $A \cup B \cup C$ (b) $A \setminus B \setminus C$ (c) $(B \cap A) \setminus C$ (d) $(A \cup B) \cap C$

Problem

Exercise 1.3.4 Interpret the following set definition (Russell's paradox) and discuss whether $X \in X$ or $X \notin X$:

$$X = \{x \mid x \notin x\} \quad (1.3)$$

Computer exercises

Exercise 1.3.5 Write a program that generates the sequence $(x_n)_{n=0}^{100}$ for $x_n = n$.

Exercise 1.3.6 Write a program that generates the odd numbers between 1 and 100.

Exercise 1.3.7 Write a program that computes the sum $\sum_{n=0}^{100} x_n$ for $x_n = n$.

}jd	õ ñ	y	9
}j	õ		10
}j	õ	<cAD	11

÷ ð ù ò ö ÷ ð=cAD ù ð

2.1 การเขียนทฤษฎี

ลองเริ่มต้นด้วยการเขียนเกี่ยวกับทฤษฎี จะเห็นว่าหัวข้อ **theorem** ยังคงเป็นภาษาอังกฤษอยู่ ถ้าต้องการเปลี่ยนไปเป็นภาษาไทยทำได้โดยการไปแก้ไขใน **class** หรืออาจจะทำเป็นรูปแบบของการแปลภาษาที่น่าจะสะดวกไว้มีเวลาจะดำเนินการต่อ ตอนนี้อยู่เป็นแบบ ภาษาอังกฤษไปก่อน

zQcrD }jd gYME . ZD5r. h กำหนดให้ P , Q และ R มีทฤษฎีบทเป็นลอจิกคือ จริง หรือ เท็จ ดังนั้นทฤษฎีบทต่อไปนี้จะเป็นจริง

$P \wedge Q$, $Q \wedge P$	$P _ Q$, $Q _ P$	(commutative laws)
$(P \wedge Q) \wedge R$, $P \wedge (Q \wedge R)$	$(P _ Q) _ R$, $P _ (Q _ R)$	(associative laws)
$P \wedge (Q _ R)$, $(P \wedge Q) _ (P \wedge R)$	$P _ (Q \wedge R)$, $(P _ Q) \wedge (P _ R)$	(distributive laws)
$\neg (P \wedge Q)$, $\neg P _ \neg Q$	$\neg (P _ Q)$, $\neg P \wedge \neg Q$	(De Morgan's laws)

÷ j เราพิสูจน์กฎแรกของเดอร์แกนและทิ้งส่วนที่เหลือไว้เป็นแบบฝึกหัด เพื่อพิสูจน์กฎดังกล่าว เราสร้างตารางความจริงและใส่ค่าที่เป็นไปได้ทั้งหมดซึ่งประกอบไปด้วย จริง หรือ เท็จ ของ P และ Q หลังจากนั้นเราได้ตารางความจริงซึ่งตารางจะประกอบไปด้วย 4 กรณี ดังนี้

$(P \wedge Q)$	$P \rightarrow Q$
T T	T T
T F	T F
F T	F T
F F	F F

จากนิยามของการดำเนินการทางด้านลอจิก เราสามารถสร้างตารางที่สมบูรณ์ได้ดังนี้

$(P \wedge Q)$	$P \rightarrow Q$
F T	T F
T T	T T
T F	T F
F T	F T
F F	F F

ตามนิยามที่ได้กำหนดไว้ตอนต้นเราสามารถพิสูจน์ให้เห็นว่าค่าประพจน์ที่กำหนดนั้นเป็นจริง (the equivalence,) ทุกเงื่อนไขที่เป็นไปได้ □

2.2 การนิยาม

ในเนื้อหาส่วนนี้เราจะพูดถึงการนิยาม หรือ definition กัน ว่าใน class นี้จะออกมาหน้าตาอย่างไร และใช้งานอย่างไรให้ดู code ประกอบไปด้วย

A rational Cauchy sequence is a rational sequence $(x_n)_{n=0}^{\infty}$ such that

$$\forall \epsilon > 0 \exists N \in \mathbb{N} : \forall m, n \geq N, |x_m - x_n| < \epsilon \tag{2.1}$$

In other words, for each (small) rational number $\epsilon > 0$ there is a (big) number N such that the distance $|x_m - x_n|$ between x_m and x_n is less than ϵ if both m and n are larger than or equal to N .

๓ ๓ ๓ ๓ ๓ ๓ ๓ ๓ ๓ ๓

Consider the equation $x^2 = 2$. It is easy to prove that this equation does not have any rational solutions. However, consider the following iteration formula:

$$x_n = \frac{x_{n-1} + 2/x_{n-1}}{2}; \tag{2.2}$$

where $n = 1; 2; 3; \dots$ and $x_0 = 1$. The resulting sequence of rational numbers quickly

approaches a number in the vicinity of $x = 1.4142135623731$:

$$\begin{aligned}
 x_0 &= 1 \\
 x_1 &= (x_0 + 2/x_0)/2 = 1.5 \\
 x_2 &= (x_1 + 2/x_1)/2 = 1.4166666666667 \\
 x_3 &= (x_2 + 2/x_2)/2 = 1.4142156862745 \\
 x_4 &= (x_3 + 2/x_3)/2 = 1.4142135623747 \\
 x_5 &= (x_4 + 2/x_4)/2 = 1.4142135623731 \\
 x_6 &= (x_5 + 2/x_5)/2 = 1.4142135623731 \\
 x_7 &= (x_6 + 2/x_6)/2 = 1.4142135623731 \\
 x_8 &= (x_7 + 2/x_7)/2 = 1.4142135623731 \\
 x_9 &= (x_8 + 2/x_8)/2 = 1.4142135623731 \\
 x_{10} &= (x_9 + 2/x_9)/2 = 1.4142135623731
 \end{aligned}$$

We will later see that this iteration, or any other equivalent iteration, defines the real number $\sqrt{2}$.

2.3 การใส่ Code

เนื่องจากหนังสือที่เขียนจะต้องมีส่วนประกอบของ code ด้วย ดังนั้นส่วนนี้จึงจะต้องทำเพิ่มเติมขึ้นมา จากที่ code เดิมได้เตรียมไว้ให้ ผลลัพธ์จะออกมาเป็นอย่างไรมาลองดูกันเลย สำหรับวิธีใช้ไปดูจากตัว source code กันนะ ไม่ขออธิบายในนี้

```

1 T]fcr{ _€]fŠ . t _f
2
3 ADI T_=]. {rT`gMD_Zd?MD_Z} h>
4   ] F ZD_gMD_Zdh
5   _ F ZD_gMD_Z} h
6   \ F ^c_D a{c 5D=c]D {QD T_=TAD_=D ]. {rT`
7   fz F _fj {Dr ct gg_2]?dh? T_{h aA€]}Š „. rT. 5ZD
8
9   a=c]f€{D {QD 5T{†TtD ^cr ]. {rT`
10  \d F 5T{^cr}. {rT`gMD_Zdh
11  \} F _fj {rT€g5T{^cr}. {rT`gMD_Z} h?dh
12
13 Icr T T_ r. _MDg] Rdh>
14   Icr V T_ r. _MDgTkD? ]h>
15       :r?=: F _fj †QDr Dg\} FF \d: T?V; h
16   Icr X T_ r. _MDgZD_gr hh>
17       fz: gTh2_ k r: X; ; F du
18       fz: gTh2_ k =: X; ; F du
19       fz: gVh2_ k r: X; ; F du
20       fz: gVh2_ k =: X; ; F du

```

```

21
22
23
24
25
26
27
28
29

```

Listing 2.1: Python example

Exercises

Ex 2.1 Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Determine the following sets.

- (a) $A \cup B$ (b) $A \setminus B$ (c) $A \cap B$ (d) $A \times B$

Ex 2.2 Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$. Determine the following sets.

- (a) $A \cup B$ (b) $A \setminus B$ (c) $A \cap B$ (d) $A \times B$

Ex 2.3 Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$. Determine the following sets.

- (a) $A \cup B \cup C$ (b) $A \setminus B \setminus C$ (c) $(B \cap A) \setminus C$ (d) $(A \cup B) \cap C$

Problem

Ex 2.4 Interpret the following set definition (Russell's paradox) and discuss whether $X \in X$ or $X \notin X$:

$$X = \{x \mid x \notin x\} \tag{2.3}$$

Computer exercises

Ex 2.5 Write a program that generates the sequence $(x_n)_{n=0}^{100}$ for $x_n = n$.

Ex 2.6 Write a program that generates the odd numbers between 1 and 100.

<c] f@r.DD=TD}j| Write a program that computes the sum $\sum_{n=0}^{100} x_n$ for $x_n = n$.

jd	Hf{tD-{E_	14
j}	sD-c_AtD-{E_	15
j	zQFA tD-{E_	16

PDZ?QDDTtc] D{D{†TQc€.] D_TMzQF {D{tQc€A tQc††Q {. frI{DA{D{†Z
ZcXZND. {{QFfZ=D} SŠc€rD A{QF {D{?Šc€†ZND{cTlcr} . {E_j qD ZñS {QDD_c
Tlcr} . {E_n S {QDD. ATDD=D5D{†DD_ {QF {D{._Atc} D_c_tD_tDZND P€ rAdt{
MD5€_ n VW{ _c{. {Z- 5ZA {D{ZND{QF M{Dt Šc€Tlcr} . {E_ . 5c€{ {QD tDD={DA
lc_?Qc† {QDZ{Dt . n†r{RD . _A. _T frD tE_c l{QDZcX} zQF {D{tQc€A=c_{T
.ZZ{Dt cl{Q. ZQ 5D. _A†tQc€A 5D†r{RD_T cl{QcrM. ZZ_M€ Mj} zQDDT_c
DDA lcr t f D=TZ=c{D_{?5€{ {QD_MQcI† crAt tQc€A} . {=Q{QZ_M€ Mj}

3.1 First section

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If

you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.2 Second section

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of

words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

3.3 Third section

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

After this fourth paragraph, we start a new paragraph sequence. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written

and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

This is the second paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

And after the second paragraph follows the third paragraph. Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

- [1] Terence Tao. - . *Zt T*. Vol. 1. Hindustan Book Agency, 2006.
- [2] Alan Turing. “On computable numbers, with an application to the Entscheidungsproblem”. In: *erc=DDATM cI{QDYc_Ac_ \ . {QD} . {F. Zs c=DŠ}42* (1936–7), pp. 230–265.

completeness of the real numbers, 7

logic algebra, 5, 9

rational Cauchy sequence, 6, 10

real numbers, 7

}‘

ดรรชนี