# Math 333 Weekly Homework 1

Juliet Traini

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## Exercise 1

Assume $n, a \in \mathbb{Z}$. If $a$ divides $n^2$, then $a$ divides $n$.

*Proof.* Assume $n, a \in \mathbb{Z}$. We can prove our prompt false using a counterexample. First, using the definition of divisibility, we can rewrite the prompt as "If $n^2 = ak$, then $n = ak$." Let's assume $n = 9$ and $a = 27$. When we plug these values into our first equation we get

$$(9)^2 = 27k, k \in \mathbb{Z}.$$

We can further solve this equation to reveal

$$(9)^2 = 27k => 81 = 27k \quad (1)$$
$$=> k = 3. \quad (2)$$

However, when we plug these values into our second equation we get

$$9 = 27k, k \in \mathbb{Z}.$$

We can further solve this equation to reveal

$$9 = 27k => k = \frac{1}{3}.$$

Under these conditions, we prove the prompt false by demonstrating that both of our $k$ values are not integers which shows that when $a$ divides $n^2$, $a$ does not necessarily divide $n$. □

## Exercise 2

If $n$ is an even integer, then $n^2$ is an even integer.

*Proof.* We wish to prove our prompt to be true. By the definition of even numbers and divisibility, we can write $n$ to be

$$n = 2k, k \in \mathbb{Z}.$$

Likewise, we can rewrite $n^2$ to be

$$n^2 = (2k)^2, k \in \mathbb{Z}.$$

Multiplying and rewriting $n^2$ we see

$$n^2 = (2k)^2 => n^2 = 4k^2 \tag{3}$$

$$=> n^2 = 2(2k^2). \tag{4}$$

We can then set $2k^2 = p, p \in \mathbb{Z}$ and rewrite the equation to look like

$$n^2 = 2(p).$$

Thus, by the definition of even numbers and divisibility, we prove that if $n$ is an even integer, then $n^2$ is an even integer. □